

Towards Cosmology With Void-Lensing





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time (years)

 2×10^8

temperature

time (years)

 2×10^8

Credit: Michael Blanton and SDSS collaboration

temperature

The Biggest Puzzle in Cosmology: the Cosmological Constant

$$S = \frac{1}{2\kappa} \int d^4 x \sqrt{-g} \left(R - 2\Lambda_B \right) + S_{matter} \left[g_{\mu\nu}, \Psi \right]$$
$$\Longrightarrow$$

$$\Lambda_{eff} = \Lambda_B + \frac{\kappa}{(2\pi)^3} \int dk \frac{1}{2} \omega^2(k) \quad (\hbar = c = 1)$$

cal observations suggest $\Lambda_{eff} \simeq 10^{-3} eV$

Cosmologie

The E-H action:

$$\mu\nu + \Lambda_{\rm B}g_{\mu\nu} = \kappa T_{\mu\nu}$$

The observed value for the cosmological constant should be

The Biggest Puzzle in Cosmology: the Cosmological Constant

By using dimensional regularization:

$$\rho_{vac} = \sum_{i} n_i \frac{m_i^4}{64\pi} \ln\left(\frac{m_i^2}{\mu^2}\right)$$

$$\Lambda_{eff} = \Lambda_B + \rho_{vac} \simeq 10^{-3} eV!!$$

Fine tuning problems usually means we don't understand something!

Only the Higgs contribution gives $\sim 10^{44} eV$

$$S_{\rm grav} = \frac{M_{Pl}^2}{2}$$

$$\delta S_{\text{grav}} = 0$$

 $\sqrt{-g}d^4x[R-2\Lambda]$

 $0 \Leftrightarrow G_{\mu\nu} = 0$

$$S_{\text{grav}} = \frac{M_{Pl}^2}{2} \int \sqrt{-g} d^4 x \left[\frac{M_{Pl}^2}{2} \int \frac{M_{Pl}^2}{2} \right] d^4 x \left[\frac{M_{Pl}^2}{2} \int \frac{M_{Pl}^2}{2} \right] d^4 x \left[\frac{M_{Pl}^2}{2} \int \frac{M_{P$$

 $\frac{\phi R}{\phi} - \frac{\omega(\phi)}{\phi} (\nabla \phi)^2 - 2V(\phi)$

 $\sqrt{-g}d^4x \left[R + f(R)\right]$

The Chameleon Force on Voids

 $\dot{x}^{\rho} + \Gamma^{(i)\rho}_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} \iff$

$$\overrightarrow{F}_{\phi} = -\frac{\beta_i}{M_{Pl}} \overrightarrow{\nabla}\phi$$

Andrius Tamosiunas et al. (2022)

Why To Test Gravity on Cosmological Scales? - Observational Reason

Credits: Psaltis, D. Testing general relativity with the Event Horizon Telescope

Why Voids?

credits: Diemer & Mansfield

Void Morphology

credits: Diemer & Mansfield

How To Count Voids: Excursion Set Formalism in a Nutshell

Sheth et al. (2004)

$$\delta(R) \equiv \delta_R^{(1)}(q) = \int d^3 x W_R(x) \delta^{(1)}(q+x)$$
$$S(R) \equiv \sigma^2(R) = \langle \delta^2(q,R) \rangle$$
$$F(>M) = 1 - \int_{-\infty}^{\delta_v} d\delta \Pi(\delta;R_0)$$
$$\bar{n}_v(M) = -\bar{\rho}_m \frac{F(>M)}{M}$$

$$d\delta\Pi(\delta;S) = \int_{-\infty}^{\delta_{\nu}} d\delta \frac{1}{\sqrt{2\pi S}} \left[e^{-\delta^2/2S} - e^{-(2|\delta_{\nu}| - \delta)^2/2S} \right] = erf$$

dM

Credits: Jessie Muin

Why Weak-Lensing?

Desjacques et al. (2018)

 $\delta_g(\boldsymbol{x},\tau) = \sum b_{\mathcal{O}}(\tau) \mathcal{O}(\boldsymbol{x},\tau)$ \bigcirc $\delta_g(\boldsymbol{x},\boldsymbol{\tau}) \simeq b_g^{(1)} \delta_m(\boldsymbol{x},\boldsymbol{\tau})$

Weak-Lensing Review

Geodesic equation + perturbations

$$\begin{aligned} \theta^{i} &= \theta^{i}_{s} + \Delta \theta^{i} \\ \Delta \theta^{i}(\theta) &= \frac{2}{c^{2}} \int_{0}^{\chi} d\chi' \Phi_{,i} \left(x \left(\theta, \chi' \right) \right) \chi' \left(1 - \psi_{ij} \right) \\ \psi_{ij} &\equiv \frac{\partial \Delta \theta^{i}}{\partial \theta^{j}} = \frac{\partial^{2}}{\partial \theta^{i} \partial \theta^{j}} \phi_{L}(\theta) = \frac{2}{c^{2}} \int_{0}^{\chi} d\chi' \Phi_{,ij} \left(x \left(\theta, \chi' \right) \right) \chi' \left(1 - \psi_{ij} \right) \\ A_{ij} &\equiv \frac{\partial \theta^{i}_{S}}{\partial \theta^{j}} = \left(\begin{array}{c} 1 - \kappa - \gamma_{1} & -\gamma_{2} \\ -\gamma_{2} & 1 - \kappa + \gamma_{1} \end{array} \right) \\ A_{ij} &= \delta_{ij} + \psi_{ij} \\ \hline \kappa &= \frac{\psi_{11} + \psi_{22}}{2} = \frac{2}{c^{2}} \int_{0}^{\chi} d\chi' \nabla^{2} \Phi \left(x \left(\theta, \chi' \right) \right) \chi' \left(1 - \frac{\chi}{\chi} \right) \\ \gamma_{1} &= -\frac{\psi_{11} - \psi_{22}}{2} \\ \gamma_{2} &= -\psi_{12} \end{aligned}$$

Weak-Lensing Review

Mean convergence within θ :

Weak-Lensing Review

Under the thin lens approximation:

$$\kappa(\boldsymbol{\theta}) = \frac{4\pi G}{c^2} \frac{\chi_1 \chi_{1s}}{\chi_s} \int_{\chi_1 - \Delta \chi/2}^{\chi_1 + \Delta \chi/2} d\chi \bar{\rho} \delta(\chi \boldsymbol{\theta}, \chi) = \frac{\Sigma(\boldsymbol{\theta})}{\Sigma_{cr}}$$

By defining

 $\Delta \Sigma(\theta) = \bar{\Sigma}(\leq$

$$\leq \theta) - \langle \Sigma \rangle (\theta)$$

$$= \Sigma_{\rm cr} \left\langle \gamma_{t,x} \right\rangle (\theta)$$

WL Voids

Amendola et al. (1998)

Void-Lensing and MG

$$S_{\rm grav} = \frac{M_{Pl}^2}{2} \int \sqrt{-g} d^4 x \left[R + f(R) \right]$$

$$\nabla^2 \Phi = \frac{16\pi G}{3} a^2 \delta \bar{\rho}_m + \frac{a^2}{6} \delta R(f_R)$$

$$\nabla^2(\Psi-\Phi)=\nabla^2\delta f_R$$

$$\nabla^2(\Psi + \Phi) = 8\pi G a^2 \delta \bar{\rho}_m$$

$$\sum_{i=1}^{n} -\sum_{i=1}^{n} M_{\odot} Dc^{-2} h]$$

Marius Cautun et al. (2018)

Void Lensing (DES)

How To Void Lensing?

Gadget simulation (Spergel)

What Voids Must Be

Toy Optimum Centering Void Finder

Everything Leads to OCVF

Theory X Simulation (3D)

ZOBOV Voids

Neyrinck (2008)

Comparison With Literature (3D) – Mark C. Neyrinck (2004)

OCVF 3D

ZOBOV

Hamaus et al (2014)

Comparison With Literature (3D) – Voivodic Et AL. (2020)

Void-Lensing Measurements Visualisation ($\sim 10^3 deg^2$, 0.1 < z < 0.3)

Void-Lensing Measurements

Void-Lensing Measurements – S/N

The Thin-Lens Approximation

DM Profile X VL Profile

DM Profile X VL Profile

ZOBOV

The Void-Lensing Model 2D-3D Connection

2D-3D Connection

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Projected field

3D field

 $\Sigma(r_{\perp} | R_{2D}, \Delta_{2D}, \Delta_{3D}) = \mathscr{F}[\delta_{3D}(R_{3D}, \Delta_{3D}) | R_{2D}, \Delta_{2D}]$

 $\Sigma(r_{\perp} | R_{2D}, \Delta_{2D}, \Delta_{3D}) = \frac{1}{N} \int dR_{3D} \frac{dn_{\nu}}{dR_{3D}} (R_{3D} | \Delta_{3D}) \times \int dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}, \Delta_{2D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}, \Delta_{2D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, \Delta_{3D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, \Delta_{3D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, \Delta_{3D}) + \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D}, \Delta_{3D}) + \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, \Delta_{3D}) dx_{\perp} d\phi P(x_{\perp} | R_{3D$

 $= dr_{\parallel} \delta^{eff}(r_{\perp}, r_{\parallel})$ $\Rightarrow \Delta \Sigma(r_{\perp}) = \bar{\Sigma}(< r_{\perp}) - \Sigma(r_{\perp})$

$$\Sigma(r_{\perp} | R_{2D}, \Delta_{2D}, \Delta_{3D}) = \frac{1}{N} \int dR_{3D} \frac{dn_{\nu}}{dR_{3D}} (R_{3D} | \Delta_{3D}) \times \int dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}, \Delta_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{$$

$$\Sigma(r_{\perp} | R_{2D}) = \int d \ln R_{3D} \frac{dn_{\nu}}{d \ln R_{3D}} \int dx_{\perp} d\phi \xi_{2D,3D}(x_{\perp}) \int dr_{\parallel} \delta_{3D}(|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D})$$

$$\frac{dn_v}{d\ln R} = \frac{f(\sigma)}{V(R)} \frac{d\ln \sigma^{-1}}{d\ln R} \quad \text{, where} \quad \sigma^2(R) \equiv \int \frac{dk}{2\pi^2} k^2 P_{mm}^L(k) \left| \tilde{W}(k \mid R) \right|^2$$

$$10^{-3}$$

$$10^{-3}$$

$$10^{-4}$$

$$10^{-4}$$

$$10^{-5}$$

$$10^{-6}$$

$$10^{-7}$$

$$10^{-7}$$

$$4$$

$$6$$

$$\Sigma(r_{\perp} | R_{2D}, \Delta_{2D}, \Delta_{3D}) = \frac{1}{N} \int dR_{3D} \frac{dn_{\nu}}{dR_{3D}} (R_{3D} | \Delta_{3D}) \times \int dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}, \Delta_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|r_{\perp} - x_{\perp}| | R_{3D})$$

$$\Delta_{2D}, \Delta_{3D}) = \frac{1}{N} \int dR_{3D} \frac{dn_{v}}{dR_{3D}} (R_{3D} | \Delta_{3D}) \times \int dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}, \Delta_{2D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D}, A_{2D}) \times \int dr_{\parallel} \delta_{3D} (|\mathbf{r}_{\perp} - \mathbf{x}_{\perp}| | R_{3D})$$

$$\Sigma(r_{\perp} | R_{2D}, \Delta_{2D}, \Delta_{3D}) = \frac{1}{N} \int dR_{3D} \frac{dn_{\nu}}{dR_{3D}} (R_{3D} | \Delta_{3D}) \times \int$$

$$\Sigma(r_{\perp} | R_{2D}) = \int d\ln R_{3D} \frac{dn_{\nu}}{d\ln R_{3D}} \int dx_{\perp} d$$

Preliminary Result (8 < R_v^{2D} < 15)

$$\Sigma(r_{\perp} | R_{2D}, \Delta_{2D}, \Delta_{3D}) = \frac{1}{N} \int dR_{3D} \frac{dn_{\nu}}{dR_{3D}} (R_{3D} | \Delta_{3D}) \times \int$$

$$\Sigma(r_{\perp} | R_{2D}) = \int d \ln R_{3D} \frac{dn_{\nu}}{d \ln R_{3D}} \int dx$$

 $dx_{\perp} d\phi P(x_{\perp} | R_{3D}, R_{2D}, \Delta_{3D}, \Delta_{2D}) \times dr_{\parallel} \delta_{3D}(|r_{\perp} - x_{\perp}| | R_{3D}, \Delta_{3D})$ \geq $x_{\perp}d\phi\xi_{2D,3D}(x_{\perp})\left[dr_{\parallel}\delta_{3D}(|\mathbf{r}_{\perp}-\mathbf{x}_{\perp}||R_{3D})\right]$

Information in the Void Profile

Starting from the 3D void profile (Voivodic et al., 2020):

$$\frac{\rho_v^{3D}(r_z, r_p \mid r_v)}{\bar{\rho}_m^{3D}} \equiv \xi^{1V} + b_v \xi_g$$

Where,

$$\xi^{1V} = \frac{1}{2} \left[1 + \tanh\left(\frac{\ln\left(\sqrt{\left(r_{\rm p}/r_{\rm v}\right)^2 + \left(r_{\rm z} - D_A(z)/r_{\rm v}\right)^2}\right) - \ln\left(r_0\right)}{s}\right] \right]$$

Conclusions and Future Perspectives • Void-Lensing can be measured with a significant S/N by future spectroscopic surveys. • Void-Lensing can be a key test of gravity on large scales since voids are the ideal environment and lensing is directly sensitive to the total matter field. • The VL modelling shows that the measured signal depends on the VIA • Cosmological constraints with VL is yet to be obtained. We've paved the avenue for it by: (i) showing the condition necessary to measure proj. Void profiles and (ii) proposing a

simple connection between what we measure and 3D voids.

Thank you for your attention!

2D-3D Connection

 $\delta_{v}^{2D}(R_{v}^{2D}) = F[\delta_{v}(R_{v}^{3D}), R_{v}^{2D}] = \left[dR_{v}^{3D} P_{R}(R_{v}^{3D}) \left[d\xi P_{\xi}(\xi \mid R_{v}^{3D}) \left[dr_{//} \delta_{v}^{3D} \left(\sqrt{\alpha r_{//}^{2} + (r_{\perp} - \xi)^{2}} \mid R_{v}^{3D} \right) \right] \right]$ $\approx \frac{1}{L} \sum_{i} \omega_{i} \int_{0}^{L} dr_{i} \delta_{v}^{3D} \left(\sqrt{(\alpha_{i} r_{i} - L/2)^{2} + r_{\perp}^{2}} | R_{v,i}^{3D} \right)$

- $R_v^{2D} \in [5,8](h^{-1}Mpc)$ $R_{v,1}^{3D} \in [5,8](h^{-1}Mpc)$ $R_{v,2}^{3D} \in [8,15](h^{-1}Mpc)$ $\omega_1 = 1.11, \ \omega_2 = 0.36,$
- $\alpha_1 = 0.59, \ \alpha_2 = 0.78$

Why Voids? - Voids As Just Another Tracers of LSS

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 $S_{\text{grav}}^{D} = \frac{M_{Pl,D}^{2}}{2} \int d^{D}x \sqrt{-g_{D}}R_{D}$

$$S_{\rm grav} = \frac{M_{Pl}^2}{2} \int \sqrt{-g} d$$

 $d^4x \left| R + \beta_1 R \nabla_{\mu} \nabla^{\mu} R \right|.$

$$S_{\rm grav} = \frac{M_{Pl}^2}{2} \int \sqrt{-g} d^4 x \left[R + f\left(\frac{1}{\Box}R\right) \right]$$

The Role of Void Radius

2D BGS

 r_p/R_v

time (years)

Dodelson and Schimdt

Voids As Just Another Tracers of LSS

D. Pelliciari et al. (2023)

The Biggest Puzzle in Cosmology: the Cosmological Constant

By using dimensional regularization:

$$\rho_{vac} = \sum_{i} n_i \frac{m_i^4}{64\pi} \ln\left(\frac{m_i^2}{\mu^2}\right)$$

Only the Higgs contribution gives $\sim 10^{44} eV$

Why Voids? - The Information in Void Counts (S. White, 1978)

 $P(X_i) \equiv$ The probability that there is a galaxy in dV_i at x_i $P(\bar{X}_i) \equiv$ The probability that there is no galaxy in dV_i at x_i $P(\Phi_0(V)) \equiv$ The probability that there is no galaxy at V

$$P(\Phi_0(V)) = P(\bar{X}_1, \dots, \bar{X}_N) = 1 - \left[\sum_{i=1,N} P(X_i) - \sum_{j < i} P(X_i X_j) + \sum_{k < j < i} P(X_i X_j X_k) - \dots\right]$$

But

$$P(X_1, \dots, X_N) = n^N \left[1 + \sum \xi^{(2)}(x_i, x_j) + \sum \xi^{(3)}(x_i, x_j, x_k) + \dots + \xi^{(N)}(x_1, \dots, x_N) \right] dV_1 \dots dV_N$$

The probability of finding a "hole" of a certain size contains the whole hierarchy of N-point functions!

Hints for Deviations From a Cosmological Constant!

T. M. C. Abbott et al. (2024)

